Dice, Dice, Dice

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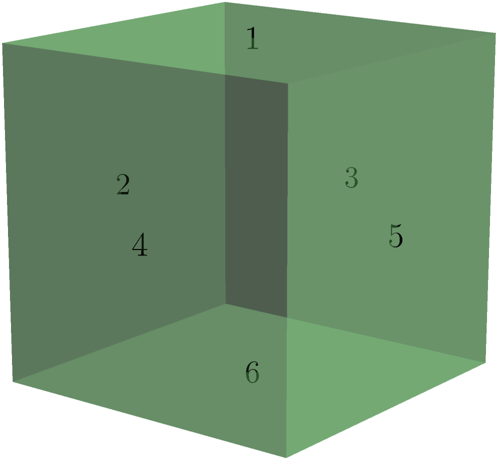
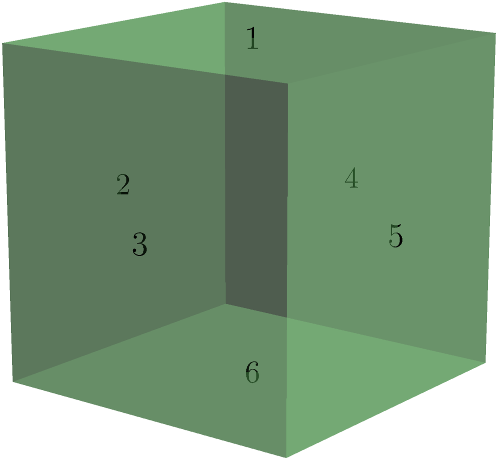
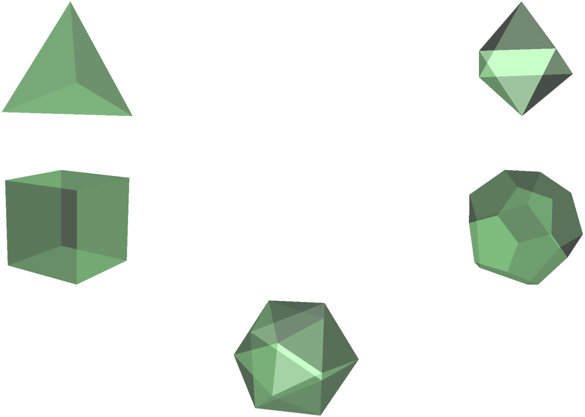
Right-handed dice



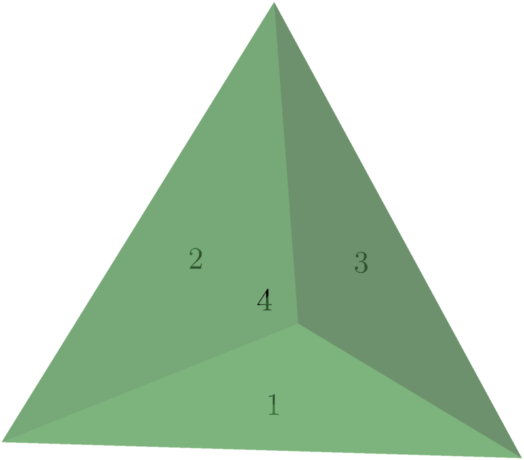
Left-handed dice



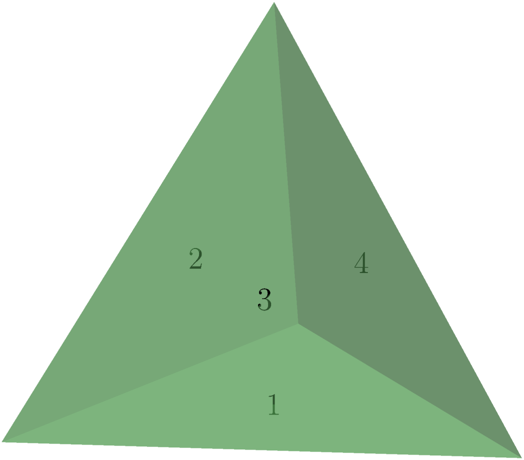
Platonic Solids



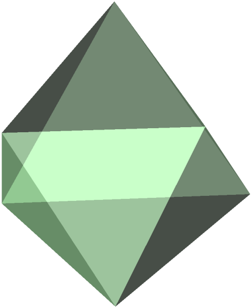
4 sided dice: tetrahedra



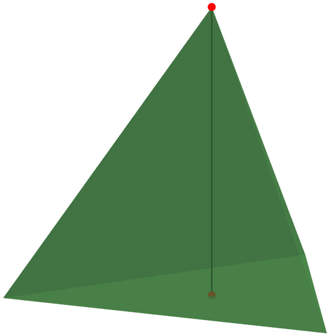
4 sided dice: tetrahedra



4 sided dice: octahedra

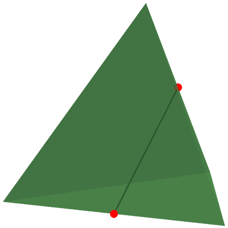


Rotational symmetries of tetrahedra



120 degree rotation through a point and a face

Rotational symmetries of tetrahedra



180 degree rotation through two edges

Rotational symmetries of tetrahedra

The number of rotations on a tetrahedron is

Rotation	Axis	Number
0° rotations	-	1
120° rotations	through a vertex and a face	4×2
180° rotations	through midpoints of edges	3

The size of the symmetry group is 12.

Rotational symmetries of octahedra

There are 4 sides, so we get

$$4 \times 3 \times 2$$

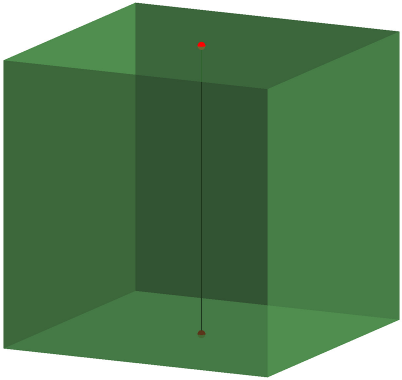
configurations.

The symmetry group has size 12.

The number of d4s is then

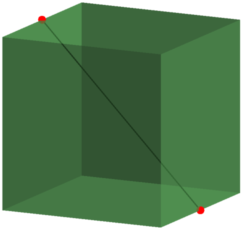
$$\frac{4 \times 3 \times 2}{12} = 2.$$

Rotational symmetries of cubes



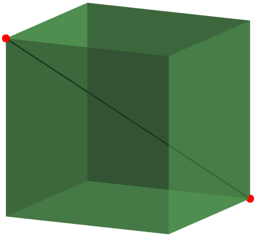
90 degree rotation through two faces

Rotational symmetries of cubes



180 degree rotation through two edges.

Rotational symmetries of cubes



120 degree rotation through two vertices.

Rotational symmetries of cubes

The number of rotations on a cube is

Rotation	Axis	Number
0° rotations	-	1
90° rotations	through centres of faces	3×2
180° rotations	through centres of faces	3
180° rotations	through midpoints of edges	6
120° rotations	through vertices	4×2

The size of the symmetry group is 24.

Rotational symmetries of cubes

There are 3 sets of opposite sides, so we get

$$2^3 \times 3 \times 2$$

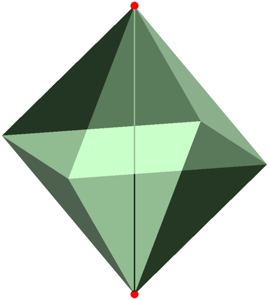
configurations.

The symmetry group has size 24.

The number of d6s is then

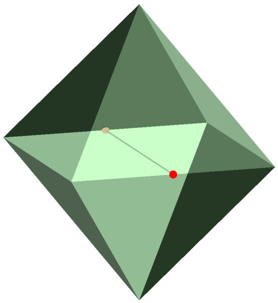
$$\frac{2^3 \times 3 \times 2}{24} = 2.$$

Rotational symmetries of octahedra



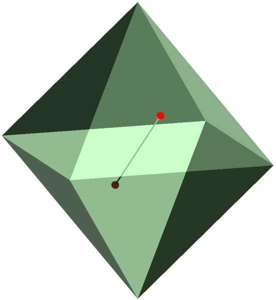
Octahedron: 90 degree rotation through two vertices.

Rotational symmetries of octahedra



Octahedron: 180 degree rotation through the midpoints of edges.

Rotational symmetries of octahedra



Octahedron: 120 degree rotation through two faces.

Rotational symmetries of octahedra

The number of rotations on octahedra is

Rotation	Axis	Number
0° rotations	-	1
120° rotations	through centres of faces	4×2
180° rotations	through midpoints of edges	6
90° rotations	through vertices	3×2
180° rotations	through vertices	3

The size of the symmetry group is 24.

Rotational symmetries of octahedra

There are 4 sets of opposite sides, so we get

$$2^4 \times 4 \times 3 \times 2$$

configurations.

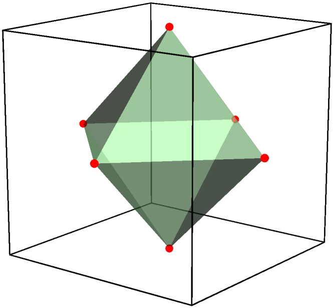
The symmetry group has size 24.

The number of d8s is then

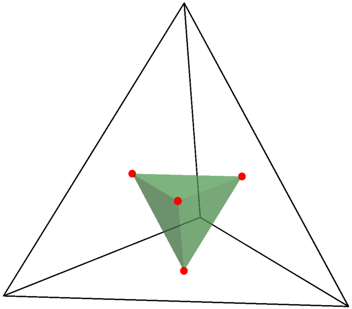
$$\frac{2^4 \times 4 \times 3 \times 2}{24} = 16.$$

Cubes and Octahedra

Notice that cubes and octahedra have the same number of rotations in their symmetry group.

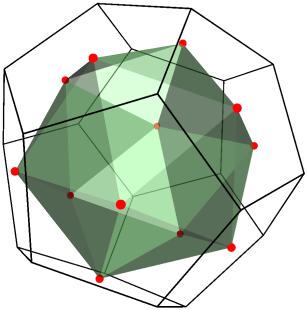


The Tetrahedron is Self-dual

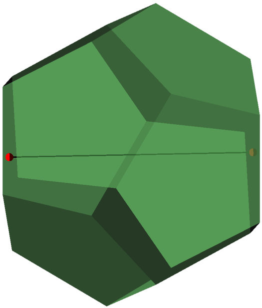


Dodecahedra and Icosahedra

Dodecahedra and icosahedra are dual solids.

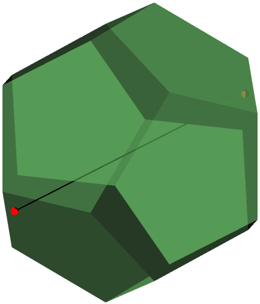


Rotational symmetries of dodecahedra



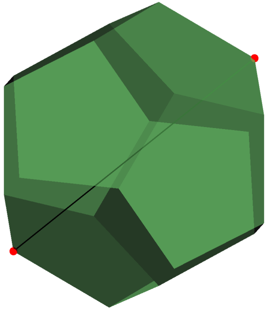
icosahedron: 72 degree rotation through two faces.

Rotational symmetries of dodecahedra



icosahedron: 180 degree rotation through two edges.

Rotational symmetries of dodecahedra



Icosahedron: 120 degree rotation through two vertices.

Rotational symmetries of dodecahedra

The number of rotations on dodecahedra is

Rotation	Axis	Number
0° rotations	-	1
72° rotations	through centres of faces	6×4
180° rotations	through midpoints of edges	15
120° rotations	through vertices	10×2

The size of the symmetry group is 60.

Rotational symmetries of dodecahedra

There are 6 sets of opposite sides, so we get

$$2^6 \times 6 \times 5 \times 4 \times 3 \times 2$$

configurations.

The symmetry group has size 60.

The number of d12s is then

$$\frac{2^6 \times 6 \times 5 \times 4 \times 3 \times 2}{60} = 768.$$

Rotational symmetries of icosahedra

There are 10 sets of opposite sides, so we get

$$2^{10} \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2$$

configurations.

The symmetry group has size 60.

Summary

Type of dice	number of configurations
d4	2
d6	2
d8	16
d12	768
d20	61 931 520

Resources

asymptote.sourceforge.net

www.math.ualberta.ca/~mroberts